

# Electromagnetic proton form factors in dual large- $N_c$ QCD: an update

B. Bisschoff <sup>(a)</sup>, C. A. Dominguez <sup>(a)</sup>, L. A. Hernandez <sup>(a)</sup>

<sup>(a)</sup> *Centre for Theoretical and Mathematical Physics, and Department of Physics, University of Cape Town, Rondebosch 7700, South Africa*

## Abstract

An updated determination is presented of the electric and magnetic form factors of the proton, in the framework of a dual-model realization of QCD in the limit of an infinite number of colours. Very good agreement with data is obtained in the space-like region up to  $q^2 \simeq -30 \text{ GeV}^2$ . In particular, the ratio  $\mu_P G_E(q^2)/G_M(q^2)$  is predicted in very good agreement with recoil polarization measurements from Jefferson Lab, up to  $q^2 \simeq -8.5 \text{ GeV}^2$ .

It has been established long ago that QCD for a large number of colours [1],  $N_c \rightarrow \infty$ , i.e.  $\text{QCD}_\infty$ , leads to a very simple hadronic spectrum, i.e. an infinite number of zero-width resonances [2]. The masses and couplings of these states, though, remain unspecified so that models are required to fix these parameters. Since in the real world  $N_c = 3$ , a naive estimate of the corrections to  $\text{QCD}_\infty$  would be at the level of 30%. However, in practice this could be an overestimate. For instance, while finite width corrections in  $\text{QCD}_\infty$  are of order  $1/N_c$ , in the hadronic sector they are only of  $\mathcal{O}(\Gamma_R/M_R)$ , where  $\Gamma_R$  and  $M_R$  are the width and the mass of a resonance, respectively. In the case of the  $\rho$ -meson this is below 20%. Furthermore,  $\text{QCD}_\infty$  models of hadron form factors in the space-like region are expected to be largely insensitive to finite-width effects in the time-like region beyond threshold. An independent argument suggesting that corrections to  $\text{QCD}_\infty$  are more likely to be at the 10% level, rather than 30%, may be found in [3]. A few models of the  $\text{QCD}_\infty$  spectrum have been proposed in the heavy-quark sector [4, 5], as well as in the light-quark sector [6].

The peculiar hadronic spectrum of  $\text{QCD}_\infty$  is reminiscent of the dual-resonance model of Veneziano [7, 8], the precursor of string theory. In the case of three-point functions, this observation motivated a specific model for the masses and couplings, Dual- $\text{QCD}_\infty$ , leading to an Euler's beta function of the Veneziano-type [9]. It should be mentioned that models of this type precede QCD. In fact, they were first proposed for the electromagnetic form factors of the proton [10, 11], the  $\Delta(1236)$  [12], and for the radiative decays of mesons [13]. They were also used in purely hadronic processes [14, 15], and  $SU(2) \times SU(2)$  chiral symmetry breaking corrections [16, 17]. Ultimately, the basic idea of a tower of radial excitations, of e.g. the  $\rho$ -meson, can be traced back to the extension of Sakurai's Vector Meson Dominance Model (VMD)[18], to Extended Vector Meson Dominance [19, 20, 21].

In this paper we update a previous determination of the proton form factors in the framework of Dual-QCD $_{\infty}$  [22] in order to account for new experimental data over an extended range of four-momentum squared, in the space-like region. In particular, new data on the ratio  $\mu_P G_E(q^2)/G_M(q^2)$ . It should be recalled that the empirical historical assumption of this ratio to be approximately constant was found in serious conflict with Jefferson Lab polarization transfer data [23, 24] up to  $q^2 \simeq -6 \text{ GeV}^2$ . Indeed, the ratio  $\mu_p G_E(q^2)/G_M(q^2)$  was found to be a monotonically decreasing function of  $q^2$ . More recent data at higher values of  $q^2$ , up to  $q^2 = -8.5 \text{ GeV}^2$ , shows a continuation of this trend.

A generic electromagnetic form factor in the framework of QCD $_{\infty}$  is given by

$$F(s) = \sum_{n=0}^{\infty} \frac{C_n}{(M_n^2 - s)}, \quad (1)$$

where  $s$  is the four-momentum squared, and the masses of the (zero-width) vector-meson resonances,  $M_n$ , as well as their couplings  $C_n$ , are unspecified and in need of a specific model. In dual-QCD $_{\infty}$  these parameters are fixed by requiring the form factors to be given by an Euler's beta function, i.e.

$$C_n = \frac{\Gamma(\beta - 1/2)}{\alpha' \sqrt{\pi}} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta - 1 - n)}, \quad (2)$$

where  $\beta$  is a free parameter determining the asymptotic behaviour of the form factor in the space-like region ( $s < 0$ ), and  $\alpha' = 1/(2 M_{\rho}^2)$  is the universal string tension entering the  $\rho$ -meson Regge trajectory

$$\alpha_{\rho}(s) = 1 + \alpha' (s - M_{\rho}^2). \quad (3)$$

The masses of the radial excitations are given by [19, 20, 21]

$$M_n^2 = M_{\rho}^2 (1 + 2n). \quad (4)$$

To compare with data, this mass formula gives for the first three radial excitations  $M_{\rho'} \simeq 1340 \text{ MeV}$ ,  $M_{\rho''} \simeq 1720 \text{ MeV}$ , and  $M_{\rho'''} \simeq 2034 \text{ MeV}$ , in reasonable agreement with the experimental values [25]  $M_{\rho'} = 1465 \text{ MeV}$ ,  $M_{\rho''} = 1720 \text{ MeV}$ , and  $M_{\rho'''} = 2000 - 2200 \text{ MeV}$ , all with very large widths,  $\Gamma \simeq 200 - 400 \text{ MeV}$ . Instead of the linear mass formula, Eq.(4), non-linear forms may be required to match the asymptotic Regge behaviour to the Operator Product Expansion (OPE) of current correlators at short distances [26]. However, the differences in the values of the masses for the first few states is only at the level of a few percent. Given that the contribution to the form factor from high mass states is factorial suppressed by the beta function, these differences have no impact in the predictions.

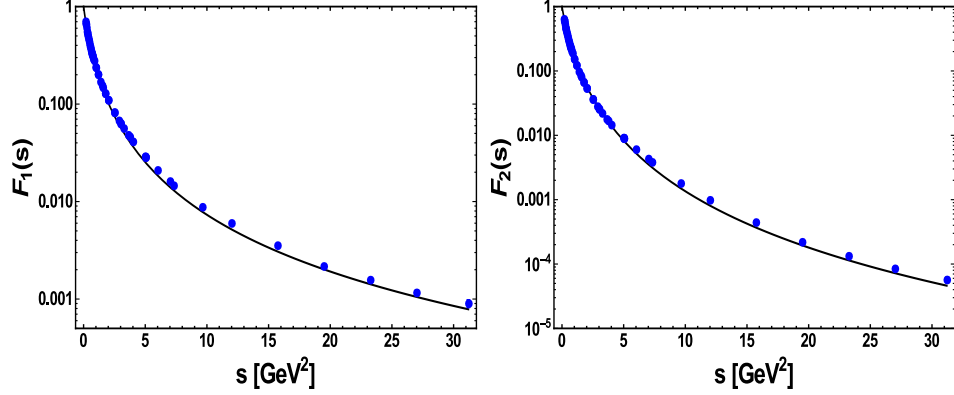


Figure 1: The form factors  $F_1(s)$  (left panel), and  $F_2(s)$  (right panel) in the space-like region as a function of  $s = -q^2$ . Dots are the experimental points obtained from inverting Eqs. (8) and (9), and using the data base compilation from [27]. Solid line is the prediction of the Dual- $QCD_\infty$  expression, Eq.(5), with  $\beta_1 = 3.105$ , and  $\beta_2 = 4.305$ . Notice the logarithmic scale.

Substituting Eqs.(2) and (4) into Eq.(1) gives the dual- $QCD_\infty$  form factor

$$\begin{aligned}
 F(s) &= \frac{1}{\sqrt{\pi}} \frac{\Gamma(\beta - 1/2)}{\Gamma(\beta - 1)} B\left(\beta - 1, \frac{1}{2} - \alpha' s\right) \\
 &= \frac{\Gamma(\beta - 1/2)}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta - 1 - n)} \frac{1}{[n + 1 - \alpha_\rho(s)]}, \quad (5)
 \end{aligned}$$

where  $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$  is the Euler Beta function. This form factor is analytic in the space-like region ( $s < 0$ ), while it has an infinite number of poles for time-like  $s$ , i.e.  $s > 0$ , and non-integer values of  $\beta$ . For integer  $\beta$  the number of poles is finite, but obviously there is no discontinuous behaviour. In fact, its imaginary part is given by

$$\text{Im } F(s) = \frac{\Gamma(\beta - 1/2)}{\alpha' \sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta - 1 - n)} \pi \delta(M_n^2 - s). \quad (6)$$

The numerical value of the single free parameter,  $\beta$ , can be determined, e.g. from a fit to data in the space-like region, or from data for the root-mean-squared radius. It has been shown for the pion electromagnetic form factor [9] that both methods produce consistent results. In the case of  $\beta = 2$ ,  $F(s)$  has only one pole, and thus it reduces to ordinary VMD.

Turning to the electromagnetic form factors of the proton, the Dirac and Pauli form factors,  $F_1(q^2)$ , and  $F_2(q^2)$  respectively, are defined as

$$\langle N(p_2) | V_\mu^{EM}(0) | N(p_1) \rangle = \bar{u}_N(p_2) \left[ F_1(q^2) \gamma_\mu + \frac{i \kappa}{2 M_N} F_2(q^2) \sigma_{\mu\nu} q^\nu \right] u_N(p_1), \quad (7)$$

where  $N$  stands for the proton,  $q^2 = (p_2 - p_1)^2$ ,  $\kappa \equiv \mu_N - 1$  and  $M_N$  are the proton's magnetic moment and mass, respectively, with  $F_{1,2}(0) = 1$ . These form factors have no kinematical singularities and satisfy dispersion relations, so that the expression Eq.(1) applies to them. In relation to electron-proton elastic scattering experiments another set of form factors, the Sach's form factors  $G_E(q^2)$  and  $G_M(q^2)$  are convenient, and defined as

$$G_E(q^2) = F_1(q^2) - \kappa \tau F_2(q^2), \quad (8)$$

$$G_M(q^2) = F_1(q^2) + \kappa F_2(q^2), \quad (9)$$

where  $\tau \equiv -q^2/4M_p^2$ , and the normalization is  $G_E(0) = 1$ ,  $G_M(0) = \mu_p$ .

Concerning the experimental data, we shall use the compilation of reanalyzed world data for  $G_E(q^2)$

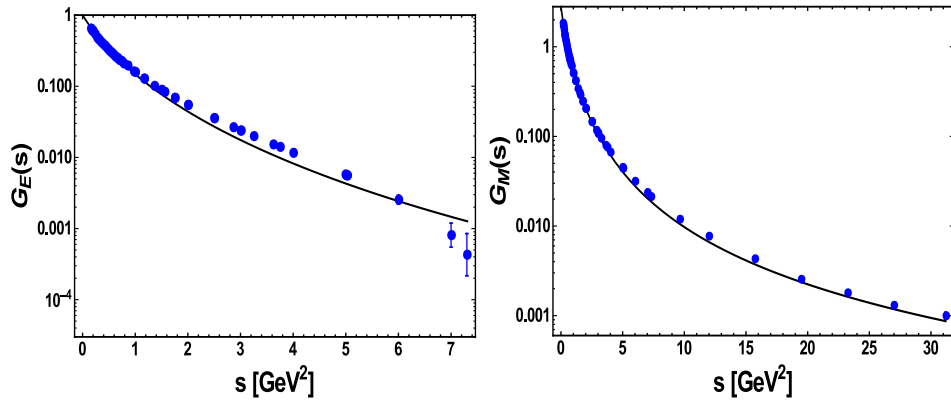


Figure 2: The form factors  $G_E(s)$  (left panel), and  $G_M(s)$  (right panel) in the space-like region as a function of  $s = -q^2$ . Dots are the experimental points using the data base compilation from [27]. Solid curves are obtained from Eqs.(8) and (9), with  $F_{1,2}(s)$  as in Fig.1. Notice the logarithmic scale.

and  $G_M(q^2)$  by Brash et al. [27], as well as more recent polarization transfer data from Jefferson Lab for  $\mu_p G_E(q^2)/G_M(q^2)$  [23, 24]. In order to find the optimal values of the free parameters  $\beta_1$  and  $\beta_2$ , we determine initial values of  $F_1(q^2)$  and  $F_2(q^2)$  for an initial set  $\beta_{1,2}$ , leading to corresponding values of  $G_E(q^2)$ ,  $G_M(q^2)$ , which are compared to the data. The process is iterated until a best fit to the latter is obtained. In this fashion we find

$$\beta_1 = 3.105, \quad (10)$$

$$\beta_2 = 4.305. \quad (11)$$

Results for the form factors  $F_{1,2}(s)$  are shown in Fig. 1, together with the data determined from that of  $G_{E,M}(s)$ . The electric and magnetic form factors in Dual-QCD $_{\infty}$ ,  $G_{E,M}(s)$ , are shown in Fig.2. Having thus determined  $\beta_{1,2}$ , the ratio  $\mu_p G_E(s)/G_M(s)$  becomes a prediction, shown in Figure 3.

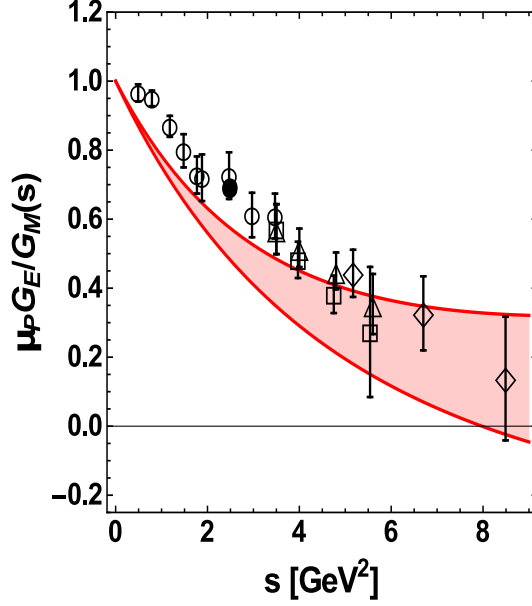


Figure 3: The prediction for the form factor ratio as a function of  $s = -q^2$ , together with polarization transfer data from Jefferson Lab [23, 24, 28, 29], using results for  $G_{E,M}(s)$  as in Fig.2. The band corresponds to keeping the parameter  $\beta_1$  fixed, and allowing  $\beta_2$  to change in the interval  $\beta_2 = 4.225 - 4.385$ . This change has a much smaller impact on the form factors themselves.

The band corresponds to keeping  $\beta_1$  fixed, and allowing  $\beta_2$  to move in the range  $\beta_2 = 4.225 - 4.385$ . Changing  $\beta_2$  in this range has very little impact on results for the form factors themselves.

Finally, we consider the various electromagnetic radii, starting with those associated with  $F_{1,2}(q^2)$ . Differentiating Eq.(5) with respect to  $q^2$ , at  $q^2 = 0$ , gives

$$\langle r_{1,2}^2 \rangle = 6 \alpha' \left[ \psi \left( \beta_{1,2} - \frac{1}{2} \right) - \psi \left( \frac{1}{2} \right) \right], \quad (12)$$

where  $\psi(x)$  is the digamma function. The resulting electric and magnetic radii, associated with  $G_E$  and  $GM$ , respectively, are

$$\langle r_M^2 \rangle^{1/2} \simeq \langle r_E^2 \rangle^{1/2} \simeq 0.8 \text{ fm}, \quad (13)$$

in agreement with current values [31, 32, 33]. It should be emphasized that the above values are the result of a one-parameter fit to each of the two form factors over a very large range of (space-like) momentum transfer. With the radius being sensitive to very low  $q^2$  data, the results above provide additional strong support for the Dual-QCD $_{\infty}$  model. If one were to restrict the form factor fit to

very low values of  $q^2$ , any increase over the result in Eq.(13) would be at the expense of the agreement between the form factors and data at high  $q^2$ .

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## References

- [1] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974).
- [2] E. Witten, Nucl. Phys. B **160**, 57 (1979).
- [3] S. Coleman, *Aspects of Symmetry, Selected Erice Lectures*, (Cambridge University Press, Cambridge, 1985), p. 351.
- [4] B. Chibisov, R. D. Dikeman, M. A. Shifman, and N. Uraltsev, Int. J. Mod. Phys. A **12**, 2075 (1997).
- [5] P. Colangelo, C. A. Dominguez, and G. Nardulli, Phys. Lett. B **409**, 417 (1997).
- [6] S. Peris, B. Phily, and E. de Rafael, Phys. Rev. Lett. **86**, 14 (2001).
- [7] G. Veneziano, Nuovo Cimento A **57**, 190 (1968).
- [8] P. H. Frampton, *Dual Resonance Models*, (Benjamin, 1974).
- [9] C. A. Dominguez, Phys.Lett. B **512**, 331 (2001).
- [10] P. H. Frampton, Phys. Rev. D **1**, 3141 (1970).
- [11] R. Iengo, and E. Remiddi, Lett. Nuovo Cimento, **18**, 922 (1969).
- [12] C. A. Dominguez, Phys. Rev. D **8**, 980 (1973).
- [13] C. A. Dominguez, Phys. Rev. D **28**, 2314 (1983).
- [14] R. A. Bryan, C. A. Dominguez, and B. J. VerWest, Phys. Rev. D **22**, 160 (1980).

- [15] C. A. Dominguez, Phys. Rev. D **27**, 1572 (1983).
- [16] C. A. Dominguez, Phys. Rev. D **7**, 1252 (1973).
- [17] C. A. Dominguez, Phys. Rev. D **16**, 2320 (1977).
- [18] J. J. Sakurai, *Currents and Mesons*, Chicago Lectures in Physics Series, University of Chicago Press (1969).
- [19] A. Bramon, E. Etim, and M. Greco, Phys. Lett. B **41**, 609 (1972).
- [20] M. Greco, Nucl. Phys. B **63**, 398 (1973).
- [21] A. Bramon, Lett. Nuovo Cimento, C **035**, 9 (2012).
- [22] C. A. Dominguez, and T. Thapedi, J. High Ener. Phys. **0410**, 003 (2004).
- [23] Jefferson Lab Hall A Collaboration, M. K. Jones, *et al.*, Phys. Rev. Lett. **84**, 1398 (2000).
- [24] Jefferson Lab Hall A Collaboration, O. Gayou *et al.*, Phys. Rev. Lett. **88**, 092301 (2002).
- [25] Particle Data Group (K. A. Olive *et al.*) Chin. Phys. **38**, 090001 (2014).
- [26] S. S. Afonin, A. A. Andrianov, V. A. Andrianov, and D. Espriu, J. High Ener. Phys. **04**, 039 (2004).
- [27] E. J. Brash, A. Kozlov, Sh. Li, and G. M. Huber, Phys. Rev. C **65**, 051001 (R).
- [28] A. J. R. Puckett *et al.*, Phys. Rev. Lett. **104**, 242301 (2010).
- [29] A. J. R. Puckett *et al.*, Phys. Rev. C **85**, 045203 (2012).
- [30] GEp2 $\gamma$  Collaboration, M. Mezziane *et al.*, Phys. Rev. Lett. **106**, 132501 (2011).
- [31] G. G. Simon, C. Schmitt, F. Borkowski, and V. H. Walther, Nucl. Phys. A **333**, 381 (1980).
- [32] J. J. Kelly, Phys. Rev. C **66**, 065203 (2002).
- [33] G. Lee, J. R. Arrington, and R. J. Hill, Phys. Rev. D **92**, 013013 (2015).